Question

A first order reaction is found to have a rate constant, $k = 5.5 \times 10^{-14}\ s^{-1}$. Find half-life of the reaction.

Answer:

For first order reaction:

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{5.5 \times 10^{-14}} = 1.26 \times 10^{13}\ s$$

Question

Show that in a first order reaction, time required for completion of 99.9% is 10 times of half-life ($t_{1/2}$) of the reaction.

Answer:

For first order reactions, rate constant k is given by:

$$k = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

When reaction is 99.9% complete, 0.1% reactants are left

i.e $a_t = \frac{0.1}{100} a_0$, $\frac{a_0}{a_t} = 1000$

Thus $k = \frac{2.303}{t_{99.9\%}} \log 1000$

For first order reaction $t_{1/2} = \frac{0.693}{k}$, $k = \frac{0.693}{t_{1/2}}$

Thus

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{t_{99.9\%}} \log 1000$$

$$t_{99.9\%} = \frac{2.303}{0.693} t_{1/2} \log 1000$$

$$= \frac{2.303}{0.693} t_{1/2} \times 3 = 10\, t_{1/2}$$

0.693

Question

Time required to decompose SO_2Cl_2 to half of its initial amount is 60 min. If the decomposition is a first order reaction, calculate the rate constant of the reaction.

Answer

For first order reaction $t_{1/2} = \frac{0.693}{k}$, so $k = \frac{0.693}{t_{1/2}}$

$= \frac{0.693}{60}$

$= 1.155 \times 10^{-2}\ min^{-1}$

Question

Calculate the half-life of a first order reaction from their rate constants given below:

i) $200\ s^{-1}$ ii) $2\ min^{-1}$ iii) $4\ years^{-1}$

Answer: For first order reaction $t_{1/2} = \frac{0.693}{k}$

i) $t_{1/2} = \frac{0.693}{200} = 3.465 \times 10^{-3}\ s$

ii) $t_{1/2} = \frac{0.693}{2} = 0.3465\ min$

iii) $t_{1/2} = \frac{0.693}{4} = 0.17325\ years$

Question

A first order reaction takes 40 min for 30% decomposition. Calculate $t_{1/2}$.

Answer:

For first order reaction, rate constant K is given by:

$$K = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

For given problem at $t = 40$ min

20% reactants are decomposed, so 70% remain, so

20% reactants are decomposed, so 70% remain, so

$$a_t = \frac{70}{100} a_0 \; , \quad \frac{a_0}{a_t} = \frac{100}{70} = \frac{10}{7}$$

Thus

$$k = \frac{2.303}{40} \log \frac{10}{7}$$

$$= \frac{2.303}{40} (\log 10 - \log 7)$$

$$= \frac{2.303}{40} (1 - 0.85)$$

$$= \frac{2.303}{40} \times 0.15 \text{ min}^{-1}$$

For first order reaction

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693 \times 40}{2.303 \times 0.15} = 80.24 \text{ min.}$$

Question

The half-life for radioactive decay of ^{14}C is 5730 years. An archaeological artifact containing wood had only 80% of the ^{14}C found in a living tree. Estimate the age of the sample.

Answer

For first order reaction, rate constant k is given by:

$$k = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

For given problem $a_t = 0.8 a_0$, $\frac{a_0}{a_t} = \frac{1}{0.8} = \frac{5}{4}$

$$\text{Thus } k = \frac{2.303}{t} \log \frac{5}{4}$$

$$= \frac{2.303}{t} (\log 5 - \log 4)$$

$$= \frac{2.303}{t} (0.7 - 0.6)$$

$$k = \frac{0.2303}{t}$$

$$t = \frac{0.2303}{k}$$

For first order reaction $t_{1/2} = \frac{0.693}{k}$, so $k = \frac{0.693}{t_{1/2}}$

$$t = \frac{0.2303}{0.693} t_{1/2} = \frac{0.2303}{0.693} \times 5730 = 1904.21 \text{ years}$$

Question

During nuclear explosion, one of the products is ^{90}Sr with half-life of 28.1 years. If 1μg of ^{90}Sr was absorbed in the bones of a newly born baby instead of calcium, how much of it will remain after 10 years and 60 years if it is not lost metabolically.

Answer:

For first order reaction, rate constant k is given by:

$$k = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

Also for first order reaction $t_{1/2} = \frac{0.693}{k}$, $k = \frac{0.693}{t_{1/2}}$

Thus

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

$$\frac{0.693}{28.1} = \frac{2.303}{t} \log \frac{a_0}{a_t}$$

$$\log \frac{a_0}{a_t} = 0.0107t$$

$$a_t = a_0 \, 10^{-0.0107t}$$

For t = 10 years

$$a_t = a_0 \, 10^{-0.0107 \times 10} = 1 \times 10^{-0.0107 \times 10} = 0.78 \mu g.$$

For t = 60 years

$$a_t = a_0 \, 10^{-0.0107 \times 60} = 1 \times 10^{-0.0107 \times 60} = 0.23 \mu g$$

For t = 60 years

$$a_t = a_0\, 10^{-0.0107\times60} = 1\times10^{-0.0107\times60} = 0.23\,\mu g$$