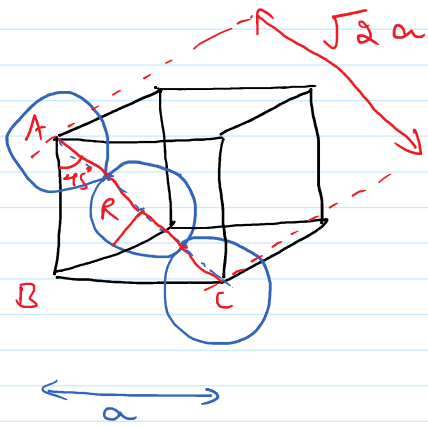
Packing efficiency

There is a lot of vacant space inbetween closely packed spheres. Packing efficiency is fraction or percentage of volume occupied by spheres w.r.t total volume of the solid

a) fcc unit cell

i) Consider a ccp or fcc lattice unit cell of edge length 'a' and radius of sphere R



ii) Spheres at corners and face centre touch each other so, $\sqrt{2}a = 4R$, $a = 2\sqrt{2}R$

iii) In fcc unit cell there are 4 spheres per unit cell.

Volume of spheres per unit cell = 4 × Volume of 1 sphere

$$= 4 \times \frac{4}{3}\pi R^3$$

$$= \frac{16}{3}\pi R^3$$

iv) Volume of unit cell = $a^3 = (2\sqrt{2}R)^3 = 16\sqrt{2}R^3$

v) Packing efficiency (η) = $\frac{\text{Volume of spheres per unit cell}}{\text{Volume of unit cell}} \times 100$

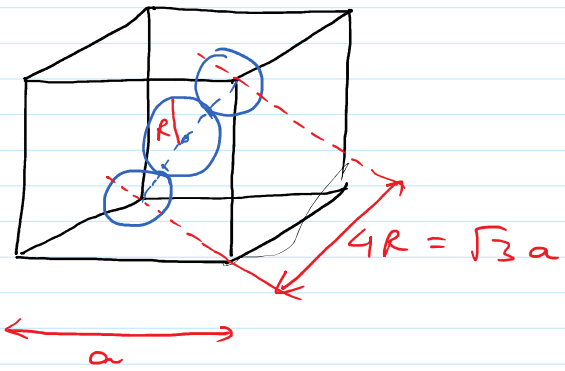
$$= \frac{\frac{16}{3}\pi R^3}{16\sqrt{2}R^3} \times 100$$

$$= 74\%$$

vi) Packing efficiency of hcp is equal to that of fcc i.e. 74%.

b) bcc unit

i) Consider a body centred cubic (bcc) lattice unit cell of edge length 'a' and radius of sphere 'R'.



ii) Spheres at corners and body centre touch each other, so $\sqrt{3}a = 4R$, $a = \frac{4}{\sqrt{3}}R$

iii) In bcc unit cell there are 2 spheres per unit cell.

iv) Volume of spheres per unit cell = 2 × Volume of 1 sphere

$= 2 \times \frac{4}{3}\pi R^3$

$= \frac{8}{3}\pi R^3$

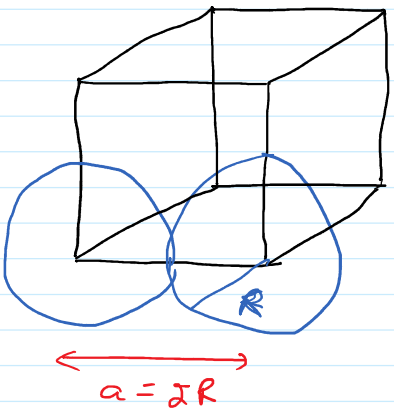
v) Volume of unit cell $= a^3 = \left(\frac{4}{\sqrt{3}}R\right)^3 = \frac{64R^3}{3\sqrt{3}}$

vi) Packing efficiency (η) = $\frac{\text{Volume of spheres per unit cell}}{\text{Volume of unit cell}} \times 100$

$= \frac{\frac{8}{3}\pi R^3}{\frac{64R^3}{3\sqrt{3}}} \times 100 = 68\%$

c) scc unit cell

i) Consider a simple cubic (scc) lattice unit cell of edge length 'a' and radius of sphere 'R'.



ii) Spheres at corners touch each other, so $a = 2R$

iii) In scc unit cell there is 1 sphere per unit cell.

Volume of spheres per unit cell = Volume of 1 sphere

$$= \frac{4}{3}\pi R^3$$

iv) Volume of unit cell $= a^3 = (2R)^3 = 8R^3$

v) Packing efficiency $(\eta) = \frac{\text{Volume of spheres per unit cell}}{\text{Volume of unit cell}} \times 100$

$$= \frac{\frac{4}{3}\pi R^3}{8R^3} \times 100$$

$$= 52.4\%$$