Question:

An element has a body-centred cubic (bcc) structure with a cell edge of 288 pm. The density of the substance is 7.2 $\frac{g}{cm^3}$. How many atoms are present in 208 g of the element?

Answer:

$$d = \frac{\text{mass of unit cell}}{\text{Volume of unit cell}} = \frac{z \times m}{a^3}$$

For bcc, $z = 2$

$$7.2 \frac{g}{cm^3} = \frac{2 \times m}{(288\ pm)^3}$$

$$7.2 \frac{g}{cm^3} = \frac{2 \times m}{(288 \times 10^{-10}\ cm)^3}$$

$1\ pm = 10^{-12}\ m$
$= 10^{-12} \times 100\ cm$
$= 10^{-10}\ cm$

$$m = 8.6 \times 10^{-23}\ g$$

Number of atoms in 208 g of the element $= \frac{208}{m}$

$$= \frac{208}{8.6 \times 10^{-23}}$$
$$= 2.42 \times 10^{24}\ \text{atoms}$$

Question

X-ray diffraction studies show that copper crystallises in an fcc unit cell with cell edge of 3.608×10^{-8} cm. In a separate experiment, copper is determined to have a density of 8.92 $\frac{g}{cm^3}$, calculate the atomic mass of copper.

Answer:

$$d = \frac{z \times M}{}$$

Answer:

$$d = \frac{z \times M}{N_A \times a^3}$$

For fcc structure, $z = 4$

$$8.92 \frac{g}{cm^3} = \frac{4 \times M}{6.022 \times 10^{23} \times (3.608 \times 10^{-8} cm)^3}$$

$$M = 63.1 \, g$$

As molar mass is 62.1 g, its atomic mass is 63.1 u.

Question

An element with molar mass $2.7 \times 10^{-2} kg \, mol^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is $2.7 \times 10^3 kg \, m^{-3}$, what is the nature of the cubic unit cell?

Ans:

$$d = \frac{z \, M}{N_A \, a^3}$$

$$2.7 \times 10^3 \frac{kg}{m^3} = \frac{2 \times 2.7 \times 10^{-2} kg \, mol^{-1}}{6.022 \times 10^{23} atoms \, mol^{-1} \times (405 \times 10^{-12} m)^3}$$

$$z = 4$$

For fcc structure $z = 4$, thus given structure has fcc unit cell.

Question

How can you determine the atomic mass of an unknown metal, if you know its density and the dimension of its unit cell? Explain

Answer:

$$d = \frac{z \times m}{a^3}$$

By knowing the values of d, a and z, value of m can be

calculated using above relation.

Question

Silver crystallises in fcc lattice. If edge length of the cell is 4.07×10^{-8} cm and density is 10.5 g cm^{-3}, calculate the atomic mass of silver.

Answer:

$$d = \frac{z \times M}{N_A \times a^3}$$

For fcc, $z = 4$

$$10.5 \frac{g}{cm^3} = \frac{4 \times M}{6.022 \times 10^{23} \times (4.07 \times 10^{-8} cm)^3}$$

$$M = 106.6 \text{ g}$$

As molar mass of silver is 106.6 g, its atomic mass is 106.6 u.

Question

Niobium crystallises in body-centred cubic structure. If density is 8.55 g cm^{-3}, calculate atomic radius of niobium using its atomic mass 93 u.

Answer:

$$d = \frac{z M}{N_A a^3}$$

For bcc structure, $z = 2$

As atomic mass of niobium is 93 u, its molar mass (M) is 93 g

$$8.55 \frac{g}{cm^3} = \frac{2 \times 93}{6.022 \times 10^{23} \times a^3}$$

$$a = 3.3 \times 10^{-8} \text{ cm}$$

For bcc unit cell radius of atom (r) is related to 'a' as: $r = \frac{\sqrt{3}}{4} a = \frac{\sqrt{3}}{4} \times 3.3 \times 10^{-8} = 1.43 \times 10^{-8}$ cm

Question

Copper crystallises into a fcc lattice with edge length 3.61×10^{-8} cm. Show that the calculated density is in agreement with its measured value of $8.92\ g\,cm^{-3}$.

Answer:

$$d = \frac{Z \times M}{N_A \times a^3}$$

for fcc, $Z = 4$, for copper $M = 63.5\ g$

$$d = \frac{4 \times 63.5}{6.022 \times 10^{23} \times (3.61 \times 10^{-8}\ cm)^3} = 8.96\ \frac{g}{cm^3}$$

Thus calculated value is very close to measured value.

Question

Aluminium crystallises in a cubic close-packed structure. Its metallic radius is 125 pm.

i) What is the length of the side of the unit cell?

ii) How many unit cells are there in $1.00\ cm^3$ of aluminium?

Answer:

i) For ccp lattice edge length (a) is related to radius of atom (r) as: $a = 2\sqrt{2}\,r$,

$$a = 2\sqrt{2} \times 125 = 353.55\ pm$$

ii) Number of unit cells in $1\ cm^3$ of aluminium

$$= \frac{1\ cm^3}{\text{Volume of unit cell}} = \frac{1\ cm^3}{a^3} = \frac{1\ cm^3}{(353.55 \times 10^{-10}\ cm)^3}$$

$$= 2.26 \times 10^{22}\ \text{unit cell}.$$

Question

Silver forms ccp lattice and x-ray studies of its

crystals show that the edge length of its unit cell is 408.6 pm. Calculate the density of silver (Atomic mass = 107.9 u)

Answer:

For ccp structure, $Z = 4$

As atomic mass of silver is 107.9 u, its molar mass is 107.9 g

$$d = \frac{Z\,M}{N_A\,a^3} = \frac{4 \times 107.9}{6.022 \times 10^{23} \times (408.6 \times 10^{-10}\ cm)^3} = 10.5\ \frac{g}{cm^3}$$